Vibration and buckling of composite structures using oscillatory radial basis functions

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Abstract

This paper addresses for the first time the analysis of laminated composite plates by oscillatory radial basis functions. These functions are very rarely used in the solution of PDEs, and this paper aims to prove that such functions can be very accurate in the vibration and buckling analysis of laminated composite plates. A radial basis function, $\phi(\|x - x_j\|)$ is a spline that depends on the Euclidian distance between distinct data centers $x_j, j = 1, 2, ..., N \in \mathbb{R}^n$, also called nodal or collocation points. The use of oscillatory radial basis functions has not been seen in the literature. This paper investigates the accuracy of such functions in the analysis of laminated composite plates.

Radial Basis Functions

The radial basis function (ϕ) approximation of a function $(\mathbf{u})$ is given by

$$\tilde{\mathbf{u}}(\mathbf{x}) = \sum_{i=1}^{N} \alpha_i \phi\left(\|x - y_i\|_2\right), \mathbf{x} \in \mathbb{R}^n \tag{1}$$

where $y_i, i = 1, .., N$ is a finite set of distinct points (centers) in $\mathbb{R}^n$. The coefficients α_i are chosen so that $\tilde{\mathbf{u}}$ satisfies some boundary conditions. The most common RBFs are

$\phi(r) = r^3$	,cubic
$\phi(r) = (1-r)_+^m p(r)$	,Wendland functions
$\phi(r) = e^{-(cr)^2}$	,Gaussian
$\phi(r) = \sqrt{c^2 + r^2}$	,Multiquadrics
$\phi(r) = (c^2 + r^2)^{-1/2}$	,Inverse Multiquadrics

where the Euclidian distance r is real and non-negative, $p(r)$ is a polinomial, and c is a shape parameter, a positive constant. In this paper we use an oscillating function, a linear Gaussian-Laguerre, defined as

$$\phi(r) = 1/\pi \; e^{-(cr)^2}\left(2 - (cr)^2\right) \tag{2}$$

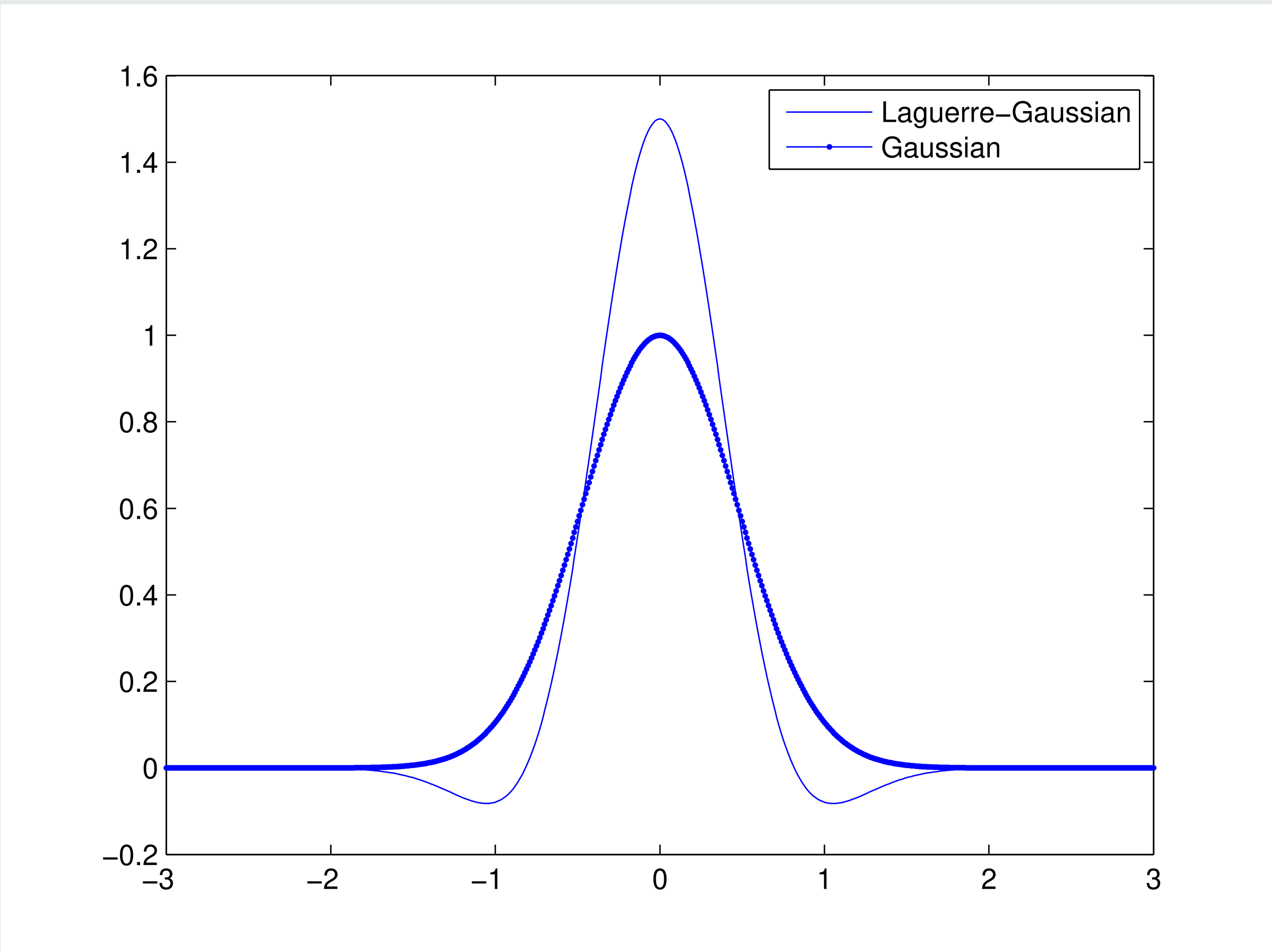

Figure 1. Oscillating and Gaussian functions

Based on the FSDT (first-order shear deformation theory), the transverse displacement $w(x, y)$ and the rotations $\theta_x(x, y)$ and $\theta_y(x, y)$ about the $y-$ and $x-$axes are independently interpolated due to uncoupling between inplane displacements and bending displacements for plates. For free vibration analysis we consider the following equations of motion:

$$D_{11}\frac{\partial^2\theta_x}{\partial x^2} + D_{16}\frac{\partial^2\theta_y}{\partial x^2} + (D_{12} + D_{66})\frac{\partial^2\theta_y}{\partial x\partial y} + 2D_{16}\frac{\partial^2\theta_x}{\partial x\partial y} + D_{66}\frac{\partial^2\theta_x}{\partial y^2} + D_{26}\frac{\partial^2\theta_y}{\partial y^2} + - kA_{45}\left(\theta_y + \frac{\partial w}{\partial y}\right) - kA_{55}\left(\theta_x + \frac{\partial w}{\partial x}\right) = I_2\frac{\partial^2\theta_x}{\partial t^2} \tag{3}$$

$$D_{16}\frac{\partial^2\theta_x}{\partial x^2} + D_{66}\frac{\partial^2\theta_y}{\partial x^2} + (D_{12} + D_{66})\frac{\partial^2\theta_x}{\partial x\partial y} + 2D_{26}\frac{\partial^2\theta_y}{\partial x\partial y} + D_{26}\frac{\partial^2\theta_x}{\partial y^2} + D_{22}\frac{\partial^2\theta_y}{\partial y^2} + - kA_{44}\left(\theta_y + \frac{\partial w}{\partial y}\right) - kA_{45}\left(\theta_x + \frac{\partial w}{\partial x}\right) = I_2\frac{\partial^2\theta_y}{\partial t^2} \tag{4}$$

$$\frac{\partial}{\partial x}\left[kA_{45}\left(\theta_y + \frac{\partial w}{\partial y}\right) + kA_{55}\left(\theta_x + \frac{\partial w}{\partial x}\right)\right] + \frac{\partial}{\partial y}\left[kA_{44}\left(\theta_y + \frac{\partial w}{\partial y}\right) + kA_{45}\left(\theta_x + \frac{\partial w}{\partial x}\right)\right] = I_0\frac{\partial^2 w}{\partial t^2} \tag{5}$$

where D_{ij} and A_{ij} are the bending and shear stiffness components, k is the shear correction factor, and I_i are the mass inertias defined as

$$I_0 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho dz, \;\; I_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho z^2 dz \tag{6}$$

Here ρ and h denote the density and the total thickness of the plate, respectively. For free vibration problems we set $p = 0$, and assume harmonic solution in terms of displacements w, θ_x, θ_y in the form

$$w(x, y, t) = W(w, y)e^{i\omega t}; \quad \theta_x(x, y, t) = \Psi_x(w, y)e^{i\omega t}; \quad \theta_y(x, y, t) = \Psi_y(w, y)e^{i\omega t} \tag{7}$$

where ω is the frequency of natural vibration.

Conclusions

In this paper we used the radial basis function collocation method to analyse buckling loads and free vibrations of isotropic and laminated plates. The oscillating radial basis functions, here used for the first time in the vibration and buckling analysis of composite plates, prove to be excellent alternative to non-oscillating functions, such as the Gaussians, and present excellent convergence and accurate results.

An example on free vibrations

The example considered is a simply supported square plate of the cross-ply lamination $[0°/90°/90°/0°]$. The thickness and length of the plate are denoted by h and a, respectively. The thickness-to-span ratio $h/a = 0.2$ is employed in the computation. The example considers a Chebyshev grid. All layers of the laminate are assumed to be of the same thickness, density and made of the same linearly elastic composite material. The following material parameters of a layer are used:

$$\frac{E_1}{E_2} = 10, 20, 30 \text{ or } 40; G_{12} = G_{13} = 0.6E_2; G_3 = 0.5E_2; \nu_{12} = 0.25$$

The subscripts 1 and 2 denote the directions normal and transverse to the fiber direction in a lamina, which may be oriented at an angle to the plate axes. The ply angle of each layer is measured from the global x-axis to the fiber direction. In all examples we use a shear correction factor $k = \pi^2/12$, as proposed in Liew.

Table 1 lists the fundamental frequency of the simply supported laminate made of various modulus ratios of E_1/E_2. It is found that the results are in very close agreement with the values of literature and the meshfree results of Liew based on the FSDT. The relative errors between the analytical and present solutions are shown in brackets. For all E_1/E_2 ratios errors are below 0.5%. Results for all E_1/E_2 ratios converge quite well.

Method	Grid	E_1/E_2			
		10	20	30	40
Liew		8.2924	9.5613	10.320	10.849
Exact (Reddy, Khdeir)		8.2982	9.5671	10.326	10.854
Present Oscillatory	9×9	8.3000	9.5413	10.2688	10.7654
	13×13	8.2999	9.5411	10.2686	10.7652
	17×17	8.2999	9.5411	10.2686	10.7652
	21×21	8.2999	9.5411	10.2686	10.7652
Present Gaussians	9×9	8.2999	9.5411	10.2686	10.7652
	13×13	8.2999	9.5411	10.2686	10.7652
	17×17	8.2999	9.5411	10.2686	10.7652
	21×21	8.2999	9.5411	10.2686	10.7652

Table 1. The normalized fundamental frequency of the simply-supported cross-ply laminated square plate $[0°/90°/90°/0°]$ $(\bar{w} = (wa^2/h)\sqrt{\rho/E_2}, h/a = 0.2)$

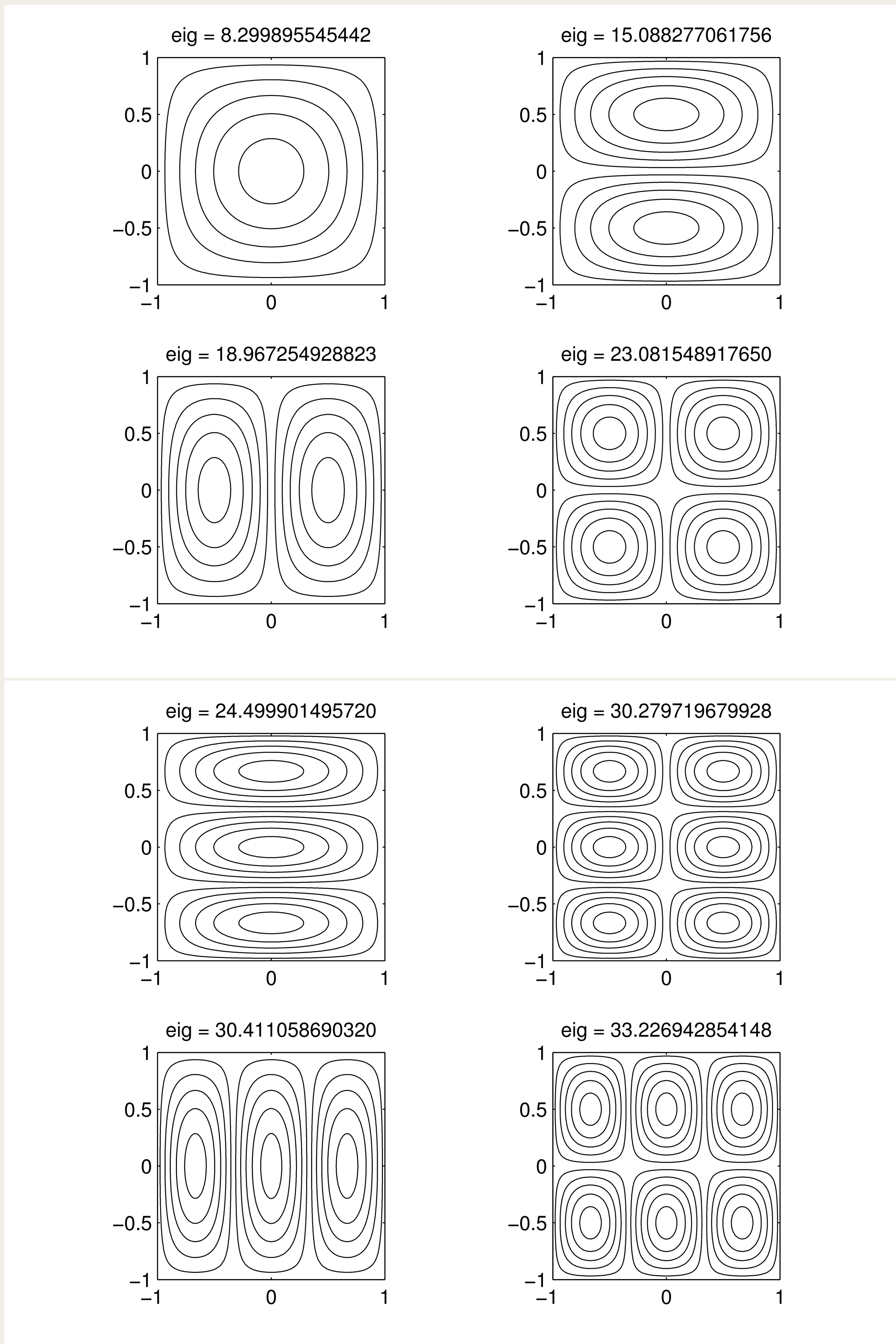

Figure 2. First eight vibration modes of the simply-supported cross-ply laminated square plate $[0°/90°/90°/0°]$, $E_1/E_2 = 10$, 13×13 nodes

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